Calculation Policy

This policy lays out our concrete, pictorial and abstract approach towards calculation.

This is based on various calculation policies, from White Rose, Cliffe VC Primary School, St Marys, Penzance and Abacus.

January 2019

Addition

YR Statutory Requirements:

Pupils should be taught to:

Birth –to 11 months – notice changes in number of objects / images , sounds in groups of and upto 3

8 – 20 months - has some understanding that things exist even when out of sight

16-26 months – Begins to organise and categorise objects -sorting

22 – 36 months – knows that a group of things changes in quantity when something is added or taken away

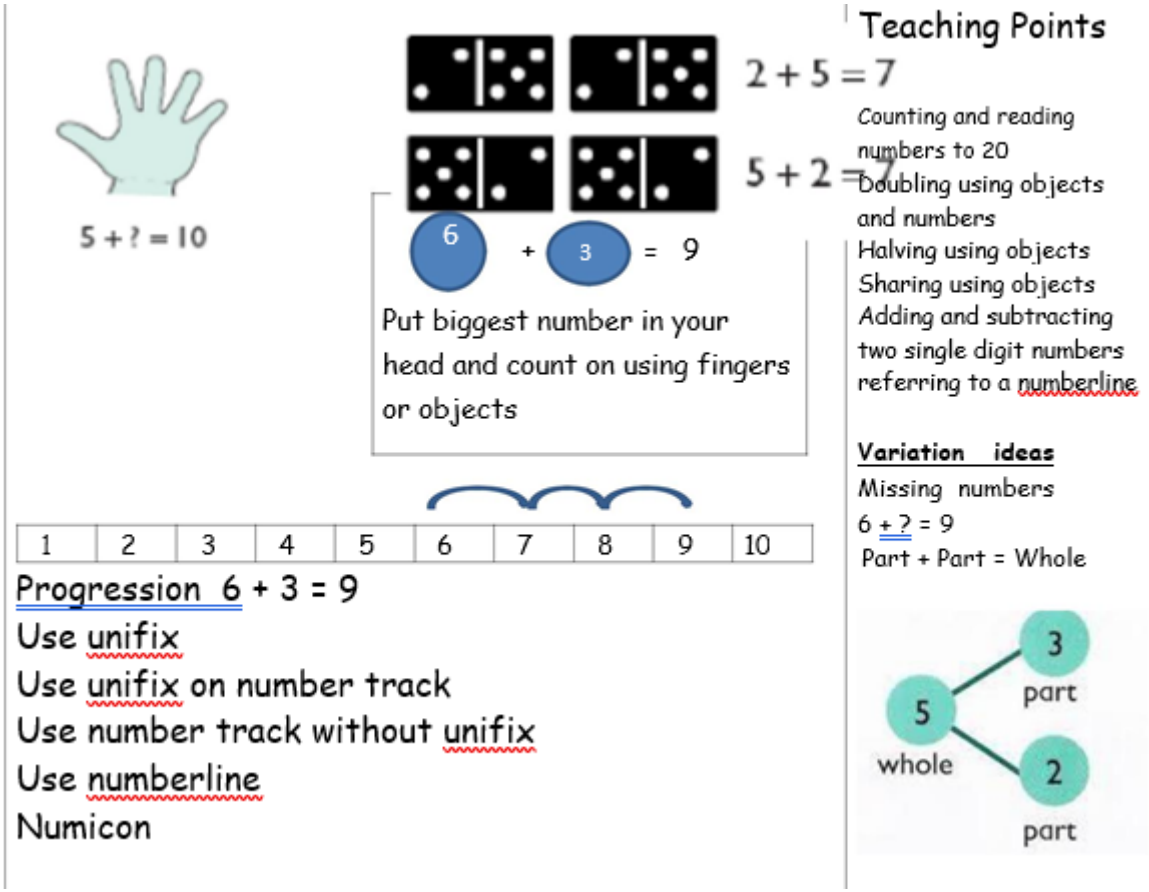
30 – 50 – separates a group of 3 or 4 objects in different ways beginning to recognise that the total is still the same

40-60 – finds the total number of items in two groups by counting all of them

In practical activities and discussions begins to use the vocabulary involved in addition and subtraction

Vocabulary

Add, more, and, make, sum, total, altogether, score, double, one more, two more, ten more…



Y1

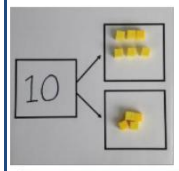
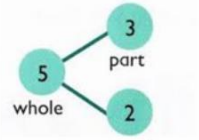
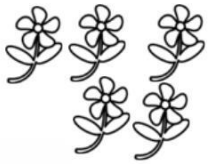
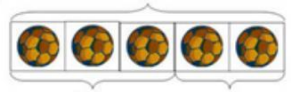
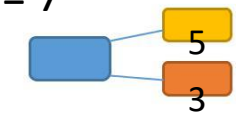
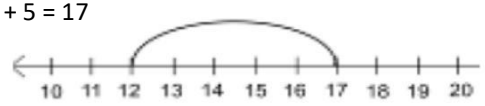
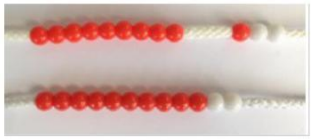
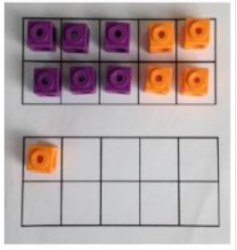
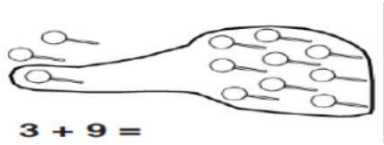
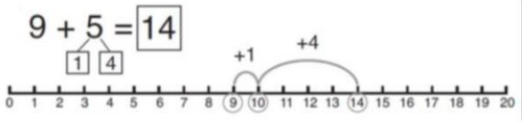
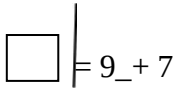
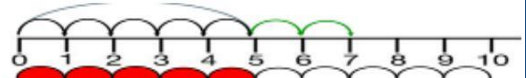
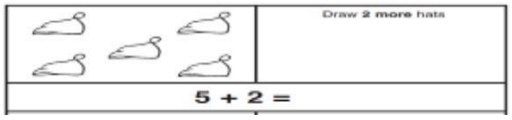
Statutory Requirements

Pupils should be taught to:

- read, write and interpret mathematical statements involving addition (+) and equals (=) signs –
- represent and use number bonds and related subtraction facts within 20
- add one-digit and two-digit numbers to 20, including zero
- solve one-step problems that involve addition, using concrete objects and pictorial representations, and missing number problems such as 9 = □ + 7.

Vocabulary

Put together, add, altogether, total, more, plus, make, sum, score, double, near double, one more, two more… ten more, how many more to make …?

Objective & Strategy	Concrete **Coat hangers, bead strings, 100 squares, case ten, numicon, counting objects.**	Pictorial	Abstract
Combining two parts to make a whole: part- whole model	   Use part part whole model. Use cubes to add two numbers together as a group or in a bar. 	    Use pictures to add two numbers together as a group or in a bar.	4 + 3 = 7  10= 6 + 4 Use the part-part whole diagram as shown above to move into the abstract.
Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.
Regrouping to make 10. *This is an essential skill for column addition later.*	6 + 5 = 11   Start with the bigger number and use the smaller number to make 10. Use ten frames.	 Use pictures or a number line. Regroup or partition the smaller number using the part part whole model to make 10. 	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now? Begin to introduce 
Represent & use number bonds and related subtraction facts within 20	 2 more than 5. Snakes and ladders game.	 	Emphasis should be on the language *'1 more than 5 is equal to 6.'* *'2 more than 5 is 7.'* *'8 is 3 more than 5.'*

Y1

Y2

Statutory Requirements

Pupils should be taught to:

- solve problems with addition:
  - using concrete objects and pictorial representations, including those involving numbers, quantities and measures
  - applying their increasing knowledge of mental and written methods

- recall and use addition facts to 20 fluently, and derive and use related facts up to 100

- add numbers using concrete objects, pictorial representations, and mentally, including:
  - a two-digit number and ones
  - a two-digit number and tens
  - two two-digit numbers
  - adding three one-digit numbers

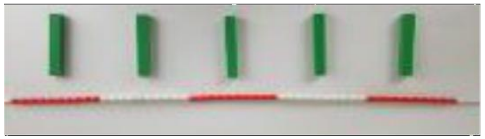
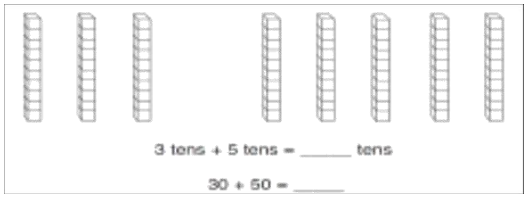
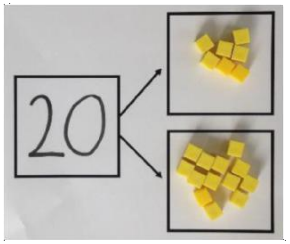
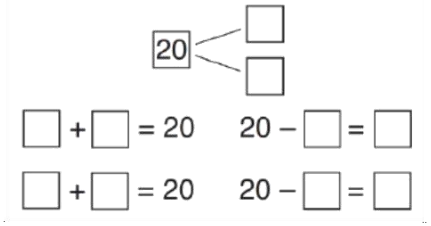
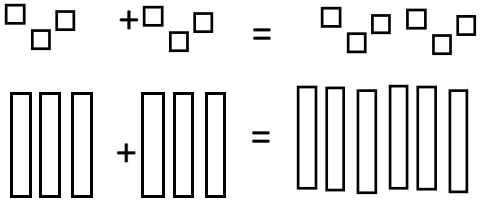
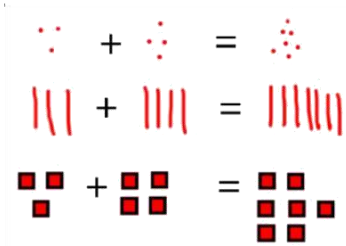
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot

- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems

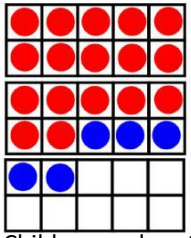
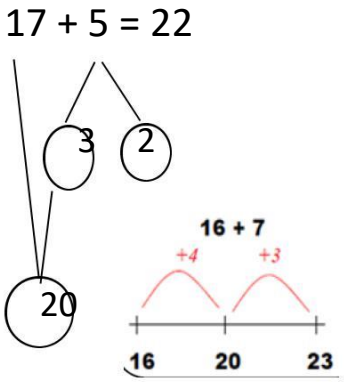
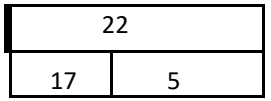
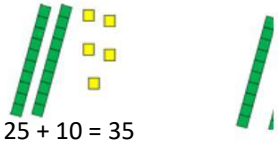
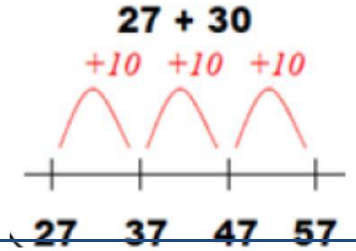
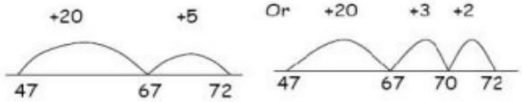
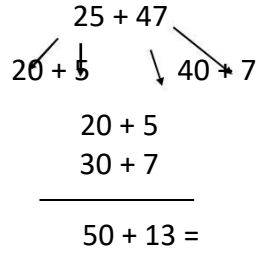
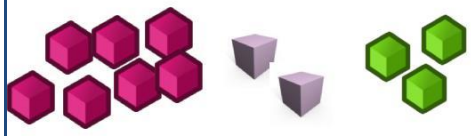
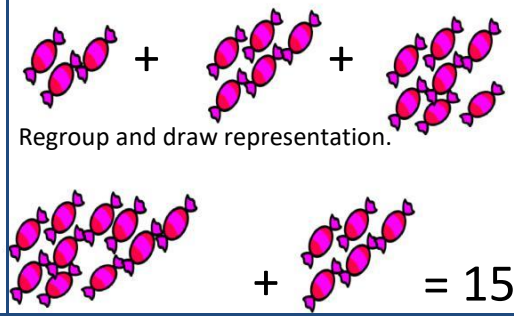
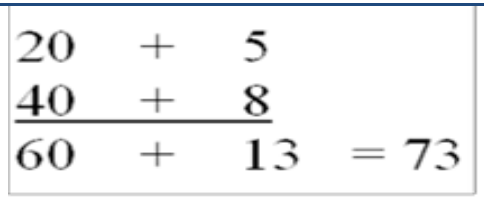
Vocabulary

Put together, add, addition, more, plus, make, sum, total, altogether, total, score, double, near double, one more, two more, ten more…, one hundred more… How many more to make…?

Y2

Objective & Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten	50= 30 = 20  Model using dienes and bead strings	 Use representations for base ten.	20 + 30 = 50 70 = 50 + 20 40 + □ = 60
Use known number facts Part part whole	 Children explore ways of making numbers within 20		□ + 1 = 16 16 – 1 = □ 1 + □ = 16 16 – □ = 1
Using known facts		 Children draw representations of H,T and O	3 + 4 = 7 *leads to* 30 + 40 = 70 *leads to* 300 + 400 = 700
Bar model	 3 + 4 = 7	 7 + 3 = 10	 23 + 25 = 48

Y2

Objective & Strategy	Concrete	Pictorial	Abstract
Add a two digit number and ones	 17 + 5 = 22 Use ten frame to make 'magic ten Children explore the pattern. 17 + 5 = 22 27 + 5 = 32	Use part part whole and number line to model. 	17 + 5 = 22 Explore related facts 17 + 5 = 22 5 + 17 = 22 22—17 = 5 22—5 = 17 
Add a 2 digit number and tens	 25 + 10 = 35 Explore that the ones digit does not change		27 + 10 = 37 27 + 20 = 47 27 + □ = 57
Add two 2-digit numbers	 Model using dienes , place value counters and numicon	 Use number line and bridge ten using part whole if necessary.	
Add three 1-digit numbers	 Combine to make 10 first if possible, or bridge 1o then add third digit		

Y3 Statutory Requirements

Pupils should be taught to:

- add numbers mentally, including:
  - a three-digit number and ones
  - a three-digit number and tens
  - a three-digit number and hundreds

- add numbers with up to three digits, using formal written methods of columnar addition

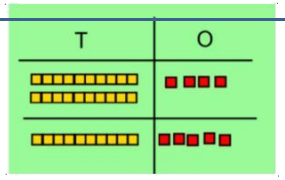
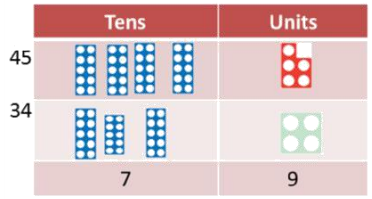
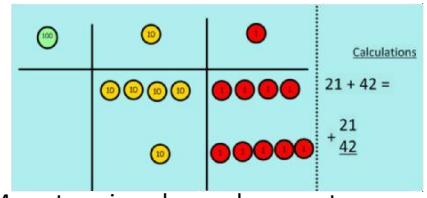
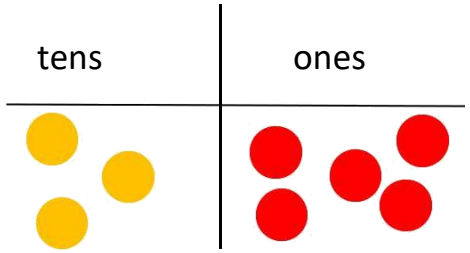
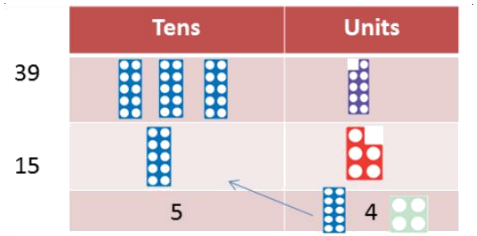
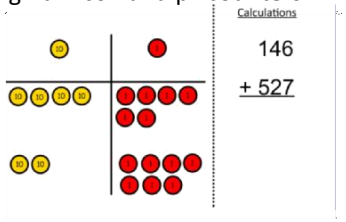
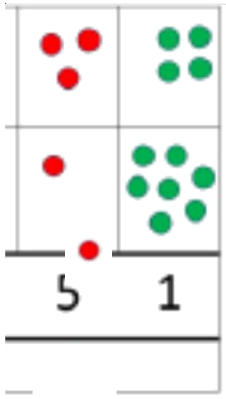
- estimate the answer to a calculation and use inverse operations to check answers

- solve problems, including missing number problems, using number facts, place value, and more complex addition.

Vocabulary

Put together, add, addition, more, plus, make, sum, total, altogether, score, double, near double, one more, two more, ten more, one hundred more, How many more to make…?

Y3

Objective & Strategy	Concrete	Pictorial	Abstract
Column Addition—no regrouping (friendly numbers) Add two or three 2 or 3-digit numbers.	 Model using Dienes or numicon Add together the ones first, then the tens.   Move to using place value counters	Children move to drawing the counters using a tens and one frame. 	223 +114 337 Add the ones first, then the tens, then the hundreds.
Column Addition with regrouping.	 Exchange ten ones for a ten. Model using numicon and pv counters. 	 Children can draw a representation of the grid to further support their understanding, carrying the ten **above** the line	Use expanded column addition where digits in a column add to more than the column value 400 60 6 300 50 8 + 100 10 800 20 4 Compact column addition with two or more 3-digit numbers or towers of 2-digit numbers 347 286 + 495 21 1128

Y4

Statutory Requirements

Pupils should be taught to:

- add with up to 4 digits using the formal written methods of columnar addition where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve addition two-step problems in contexts, deciding which operations and methods to use and why.

Y5

- add whole numbers with more than 4 digits, including using formal written methods (columnar addition)
- add numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition multi-step problems in contexts, deciding which operations and methods to use and why.

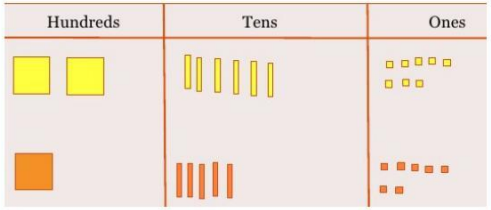
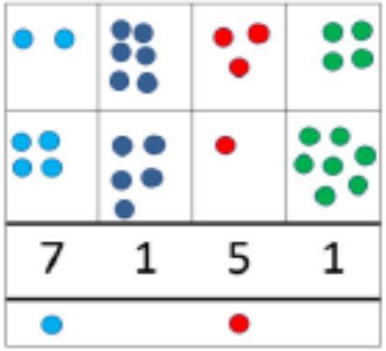
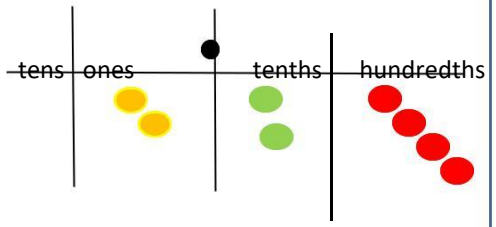
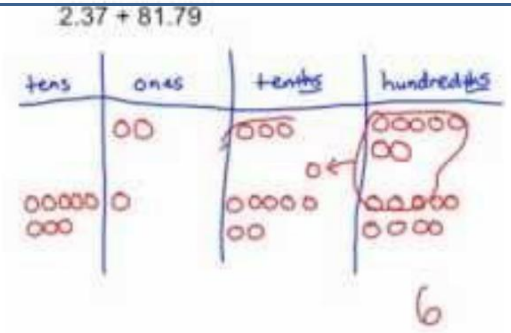
Y6

- solve addition multi-step problems in contexts, deciding which operations and methods to use and why

Vocabulary

Put together, add, addition, more, plus, sum, total, altogether, scored, double, near double, How many more to make…?

Y4-6

Objective & Strategy	Concrete	Pictorial	Abstract
Y4—add numbers with up to 4 digits	Children continue to use dienes or pv counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand. 	 Draw representations using pv grid.	Build on expanded column addition to develop compact column addition with larger numbers e.g. *1466 + 4868* 1000 400 60 6 4000 800 60 8 + 1000 100 10 6000 300 30 4 Compact column addition with larger numbers e.g. *5347 + 2286 + 1495* 5347 2286 + 1495 121 9128 Use expanded and compact column addition to add amounts of money
Y5—add numbers with more than 4 digits. Add decimals with 2 decimal places, including money.	As year 4  Introduce decimal place value counters and model exchange for addition.		Expanded column addition for money leading to compact column addition for adding several amounts of money e.g. *£14·64 + £28·78 + £12·26* £14 60p 4p £28 70p 8p + £12 20p 6p £1 10p £55 60p 8p Use compact addition to add decimal numbers with up to 2 decimal places e.g. *15·68 + 27·86* 15·68 + 27·86 11·1
Y6—add several numbers of increasing complexity Including adding money, measure and decimals with different numbers of decimal points.	As Y5	As Y5	Compact column addition with money e.g. *£14·64 + £28·78 + £12·26* £14·64 + £28·78 £12·26 11·1 £55·68 Insert zeros for place holders.

Subtraction

YR Statutory Requirements:

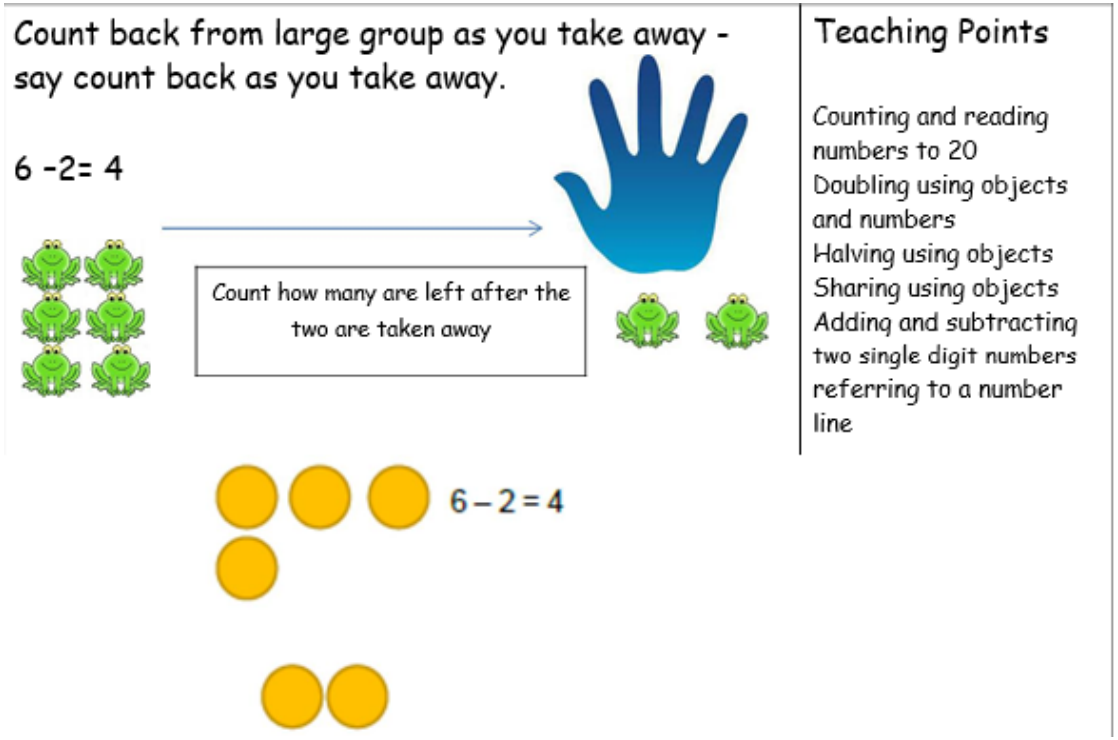
Pupils should be taught to:

Birth –to 11 months – notice changes in number of objects / images , sounds in groups of and upto 3
8 – 20 months - has some understanding that things exist even when out of sight
16-26 months – Begins to organise and categorise objects -sorting
22 – 36 months – knows that a group of things changes in quantity when something is added or taken away
30 – 50 – separates a group of 3 or 4 objects in different ways beginning to recognise that the total is still the same
40-60 – Understands subtraction as taking away objects from a group and counting on how many are left.
In practical activities and discussions begins to use the vocabulary involved in addition and subtraction

Early Learning Goal Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.

Vocabulary

Take (away), leave, how many left/left over?,
how many are gone? One less. Two less… ten less…?
how many fewer is… than…?
Difference between is the same as…



Y1

Statutory Requirements

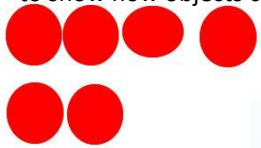
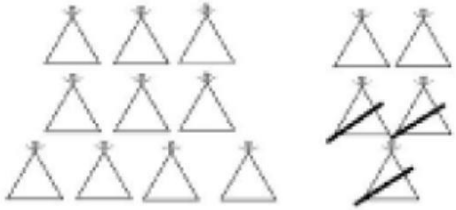
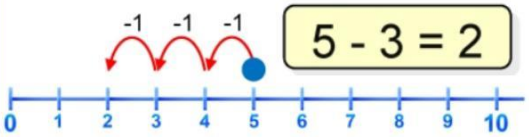
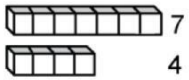
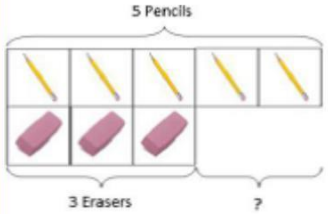
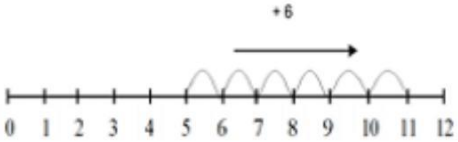
Pupils should be taught to:

- read, write and interpret mathematical statements involving subtraction (-) and equals (=) signs
- represent and use number bonds and related subtraction facts within 20
- subtract one-digit and two-digit numbers to 20, including zero
- solve one-step problems that involve subtraction, using concrete objects and pictorial representations, and missing number problems such as 9 = □ - 7.

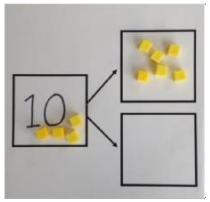
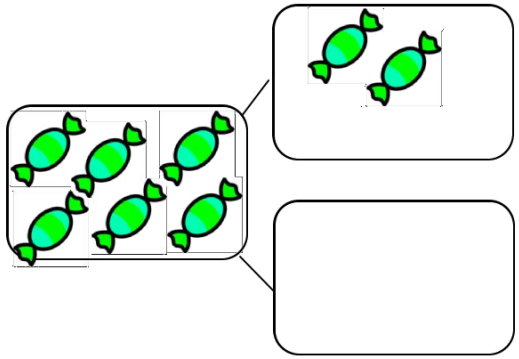
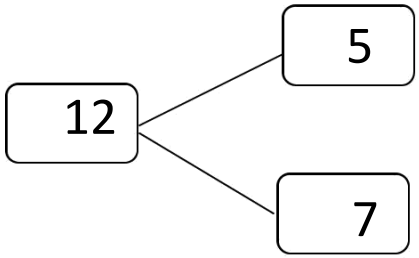
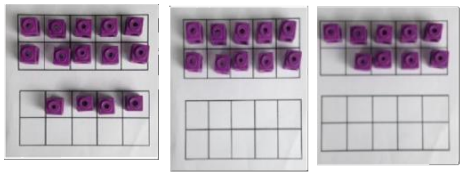
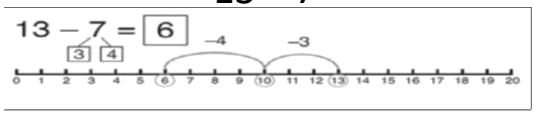
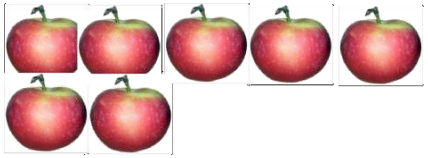
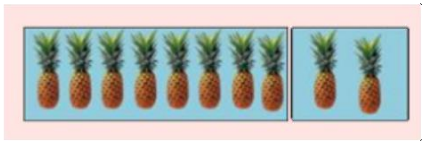
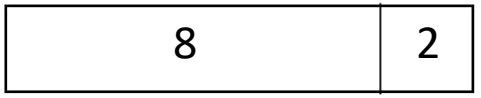
Vocabulary

How many left/left over?, How many gone? One less, ten less, how many fewer is…than…? How much less is….? Difference between, half, halve, = equals, is the same as, How many more is… than? How much more is….? Subtract, take away, minus, leave.

Y1 SUBTRACTION

Objective & Strategy	Concrete	Pictorial	Abstract
Taking away ones.	Use physical objects, counters , cubes etc to show how objects can be taken away.  6—4 = 2  4—2 = 2	 15 – 3 = 12 Cross out drawn objects to show what has been taken away.	7—4 = 3 16—9 = 7
Counting back	 Move objects away from the group, counting backwards.  Move the beads along the bead string as you count backwards.	 Count back in ones using a number line.	Put 13 in your head, count back 4. What number are you at?
Find the Difference	Compare objects and amounts  'Seven is 3 more than four' 'I am 2 years older than my sister'  Lay objects to represent bar model.	Count on using a number line to find the difference. 	Hannah has12 sweets and her sister has 5. How many more does Hannah have than her sister.?

Y1 SUBTRACTION

Objective & Strategy	Concrete	Pictorial	Abstract
Represent and use number bonds and related subtraction facts within 20 Part Part Whole model	 Link to addition. Use PPW model to model the inverse. If 10 is the whole and 6 is one of the arts, what s the other part? 10—6 = 4	 Use pictorial representations to show the part.	Move to using numbers within the part whole model. 
Make 10	14—9  Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.	13—7  Jump back 3 first, then another 4. Use ten as the stopping point.	16—8 How many do we take off first to get to 10? How many left to take off?
Bar model	 5—2 = 3		 10 = 8 + 2 10 = 2 + 8 10—2 = 8 10—8 = 2

Y2 Statutory Requirements

Pupils should be taught to:

- solve problems with subtraction:
  - using concrete objects and pictorial representations, including those involving numbers, quantities and measures
  - applying their increasing knowledge of mental and written methods
- recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100
- subtract numbers using concrete objects, pictorial representations, and mentally, including:
  - a two-digit number and ones
  - a two-digit number and tens
  - two two-digit numbers

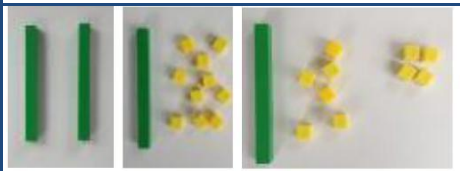
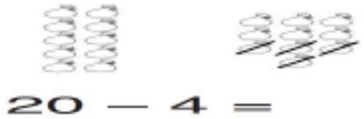
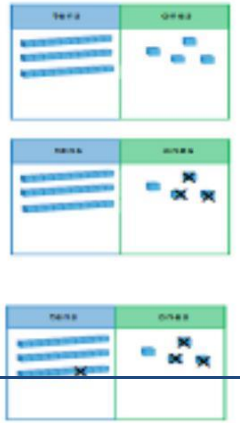
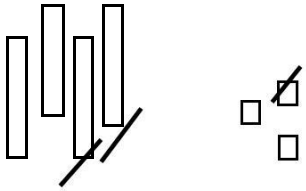
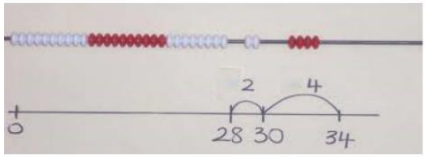
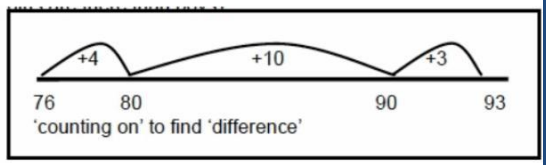
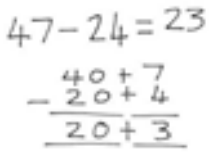
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot

- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems

Vocabulary

How many left/left over?, How many gone? One less, ten less, one hundred less, how many fewer is…than…? How much less is….? Difference between, half, halve, = equals, is the same as, How many more is… than? How much more is….? Subtract, take away, minus, leave, tens boundary.

Y2 SUBTRACTION

Objective & Strategy	Concrete	Pictorial	Abstract
Regroup a ten into ten ones	 Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'		20—4 = 16
Partitioning to subtract without regrouping. *'Friendly numbers'*	34—13 = 21  Use Dienes to show how to partition the number when subtracting without regrouping.	Children draw representations of Dienes and cross off.  43—21 = 22	43—21 = 22
Make ten strategy *Progression should be* *crossing one ten, crossing more than one ten, crossing the hundreds.*	 34—28 Use a bead bar or bead strings to model counting to next ten and the rest.	 Use a number line to count on to next ten and then the rest.	93—76 = 17 Charlie to add what she does here. 

Y3

Statutory Requirements

Pupils should be taught to:

- subtract numbers mentally, including:
- a three-digit number and ones
- a three-digit number and tens
- a three-digit number and hundreds

- subtract numbers with up to three digits, using formal written methods of columnar subtraction

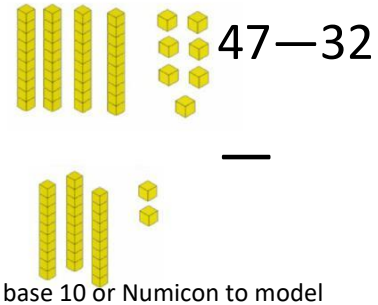
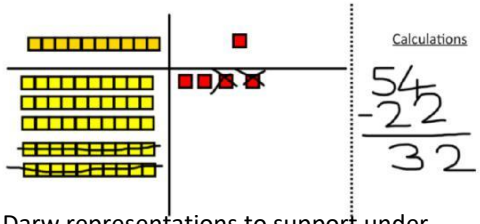
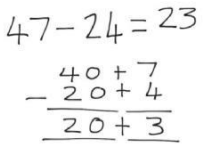
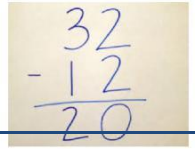
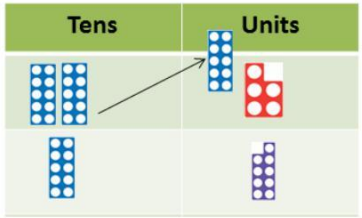
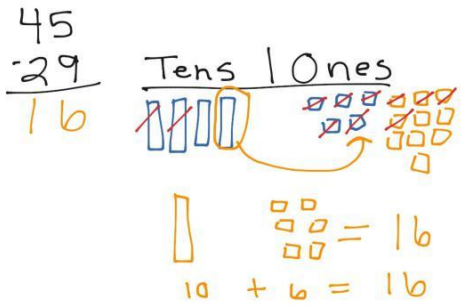
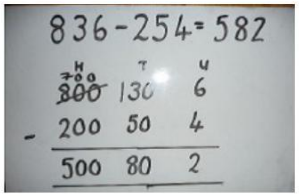
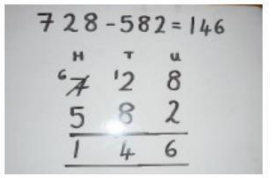
- estimate the answer to a calculation and use inverse operations to check answers

- solve problems, including missing number problems, using number facts, place value, and more complex subtraction.

Vocabulary

How many left/left over?, How many gone? One less, ten less, one hundred less, how many fewer is…than…? How much less is….? Difference between, half, halve, = equals, is the same as, How many more is… than? How much more is….? Subtract, take away, minus, leave, tens boundary.

Y3 SUBTRACTION

Objective & Strategy	Concrete	Pictorial	Abstract
Column subtraction without regrouping (friendly numbers)	 Use base 10 or Numicon to model	 Darw representations to support understanding	 Intermediate step may be needed to lead to clear subtraction understanding. 
Column subtraction with regrouping	 Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into tten ones. Use the phrase 'take and make' for exchange.	 Children may draw base ten or PV counters and cross off.	 Begin by partitioning into pv columns  Then move to formal method if ready. In Y4 they will go back to partitioned numbers.

Y4 Statutory Requirements

- **Pupils should be taught to:**

- subtract with up to 4 digits using the formal written methods of columnar subtraction where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve subtraction two-step problems in contexts, deciding which operations and methods to use and why.

Y5

Pupils should be taught to:

- subtract whole numbers with more than 4 digits, including using formal written methods (columnar subtraction)
- subtract numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

Y6

- **Pupils should be taught to:**

- solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why

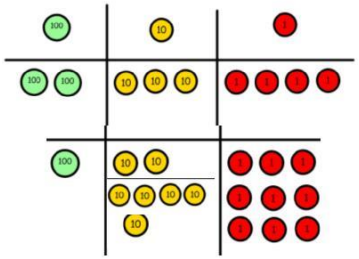
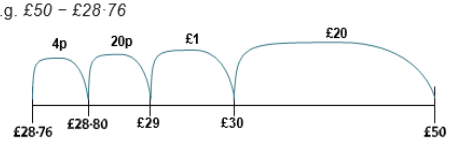
Vocabulary

How many left/left over?, How many gone? One less, ten less, one hundred less, decrease, how many fewer is…than…? How much less is….? Difference between, half, halve, = equals, is the same as, How many more is… than? How much more is….? Subtract, take away, minus, leave, tens boundary, inverse

Y5- units boundary, tenths boundary

Y6- amount, brackets, change, complements, currency, discount, exact, exactly, most least significant digit

Y4-6 SUBTRACTION

Objective & Strategy	Concrete	Pictorial	Abstract
Subtracting tens and ones Year 4 subtract with up to 4 digits. *Introduce decimal subtraction through context of money*	234 - 179  Model process of exchange using Numicon, base ten and then move to PV counters.	Children to draw pv counters and show their exchange—see Y3 Until secure with exchange, only one exchange per calculation is required. Note that even when there has been a physical exchange that the value of the number remains the same. Model written method alongside practical examples initially.	Expanded column subtraction with 3- and 4-digit numbers e.g. *726 – 358* 600 110 16 ~~700~~ ~~20~~ ~~6~~ – 300 50 8 300 60 8 Begin to develop compact column subtraction e.g. *726 – 358* 6 11 16 ~~7~~ ~~2~~ ~~6~~ – 3 5 8 3 6 8 Use phrase 'take and make' for ex- Change
Year 5- Subtract with at least 4 digits, including money and measures. *Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal*	As Year 4- informal jottings etc and estimation.	Children to draw pv counters and show their exchange—see Y3 Continue to use counting up subtraction for subtractions involving money, including finding change e.g. *£50 – £28·76*  	Compact column subtraction for numbers with up to 5 digits e.g. *16 324 – 8516* 0 15 13 1 14 ~~1~~ ~~6~~ ~~3~~ ~~2~~ ~~4~~ – 8 5 1 6 7 8 0 8
Year 6—Subtract with increasingly large and more complex numbers and decimal values.	Apply learning to combine other areas: 632,465 + (745,676 – 325,534) = 8,675,509 – (9,645,253 – 2,867,675)= 12-2.736 35.712-8.653		Compact column subtraction for large numbers e.g. *34 685 – 16 458* 2 14 7 15 ~~3~~ ~~4~~ 6 ~~8~~ ~~5~~ – 1 6 4 5 8 1 8 2 2 7

MULTIPLICATION X

YR

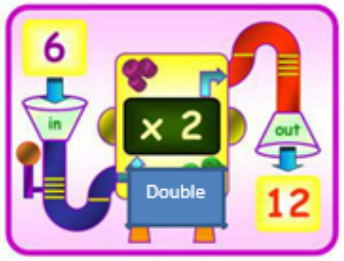
Pupils should be taught to:

Early Learning Goal Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.

Vocabulary

double, group, set, match, sort, same, different, 'clever' counting in 2s.

Doubling machines, ladybirds with spots , symmetry, dice, making it clear that doubling is adding the same number. Doubling songs



6 + 6 = 12 - Making it clear you are adding the same number.

Draw pictures to show how to double a number.

Double 4 is 8

Teaching Points

Doubling using objects and numbers

Y1

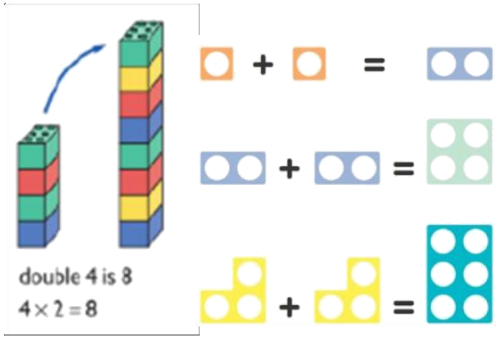
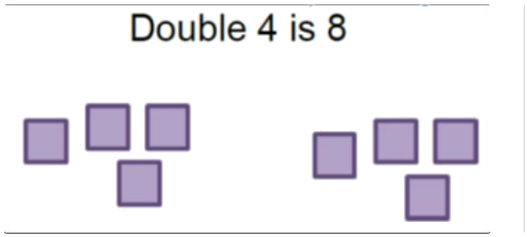
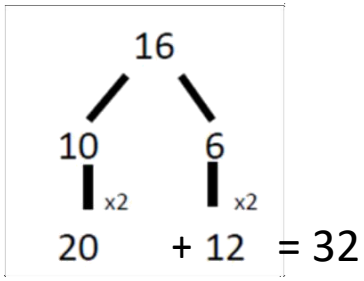
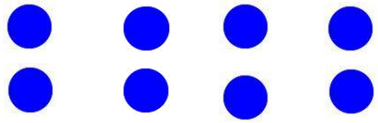
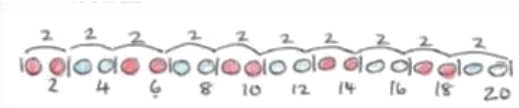
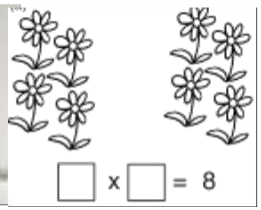
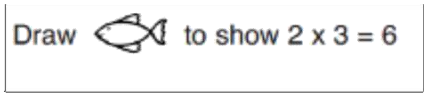
Pupils should be taught to:

• solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

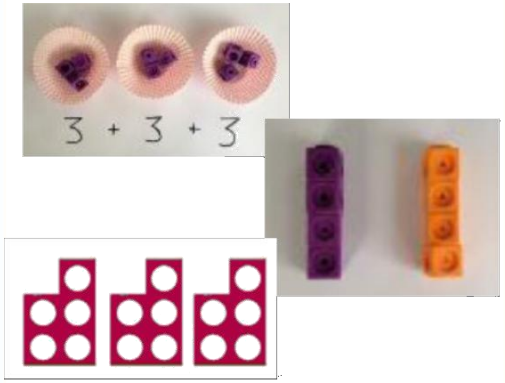
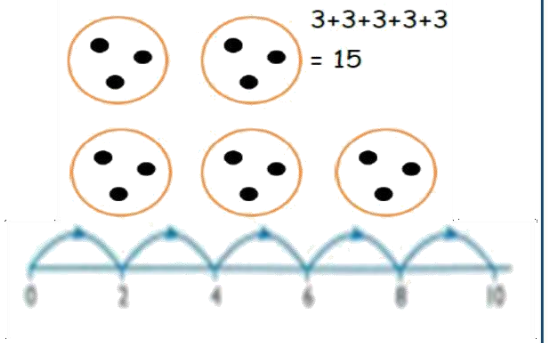
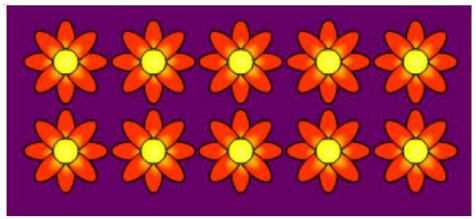
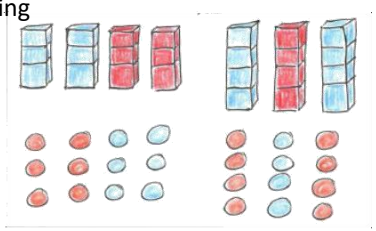
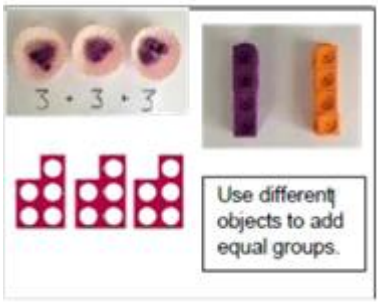
Vocabulary

lots of, groups of x, times, multiply, multiplied by, once, twice, three times, four times, five times... ten times..., times as (big, long, wide and so on), repeated addition, array, row, column, double, halve, share, share equally, one each, two each, three

MULTIPLICATION

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Use practical activities using manip-ultives including cubes and Numicon to demonstrate doubling 	Draw pictures to show how to double numbers 	Partition a number and then double each part before recombining it back together. 
Counting in multi-ples	Count the groups as children are skip counting, children may use their fin-gers as they are skip counting.  	 Children make representations to show counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of num-bers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30
Making equal groups and counting the total	  Use manipulatives to create equal groups.	 Draw and make representations	2 x 4 = 8

Y1 – MULTIPLICATION X

Objective & Strategy	Concrete	Pictorial	Abstract
Repeated addition	 Use different objects to add equal groups	Use pictorial including number lines to solve problems There are 3 sweets in one bag. How many sweets are in 5 bags altogether? 	Write addition sentences to describe objects and pictures. 
Understanding arrays	Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc. 	Draw representations of arrays to show understanding 	3 x 2 = 6 2 x 5 = 10
			

Y2

Pupils should be taught to:

- recall and use multiplication facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (×) and equals (=) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot

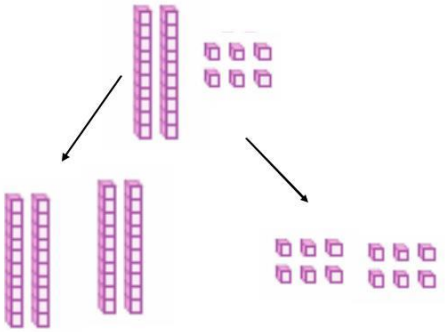
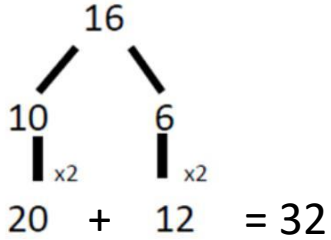
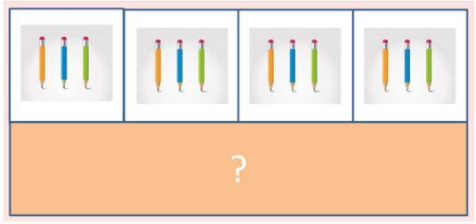
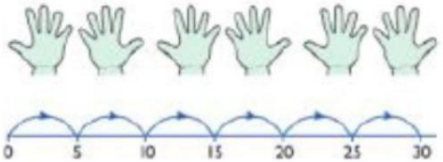
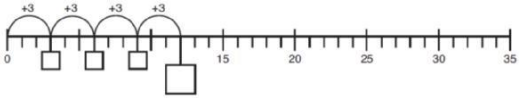
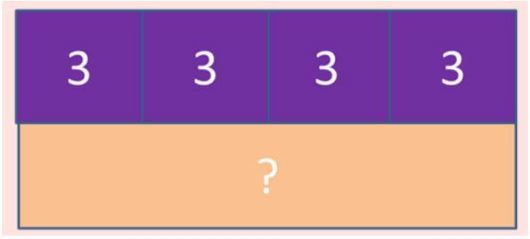
solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts

Pupils recall and use **2x 5x 10x and 3x table**
but use doubling to progress onto 4x

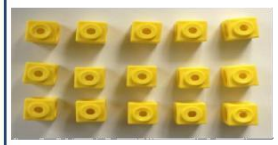
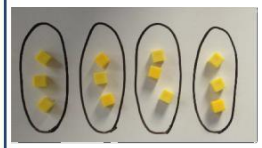
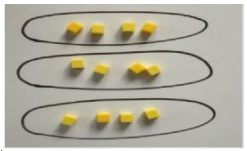
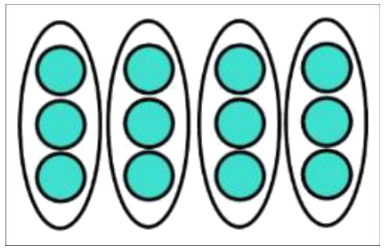
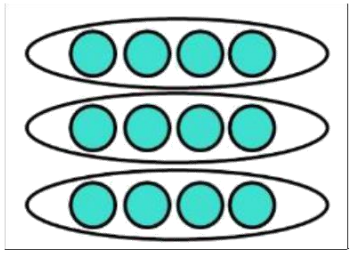
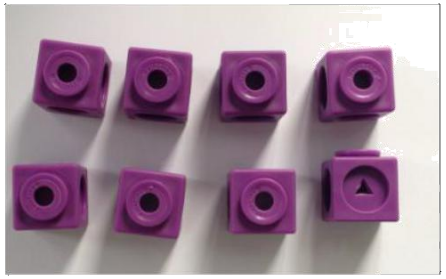
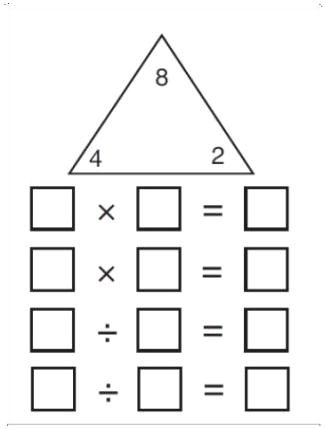
Vocabulary

lots of, groups of, x, times, multiply, multiplied by, multiple of, once, twice, three times, four times, five times... ten times...
times as (big, long, wide and so on), repeated addition, array, row, column, double, halve, share, share equally
one each, two each, three each... group in pairs, threes... tens, equal groups of ÷, divide, divided by, divided into, left, left over

Y2 MULTIPLICATION

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Model doubling using dienes and PV counters.  40 + 12 = 52	Draw pictures and representations to show how to double numbers	Partition a number and then double each part before recombining it back together. 
Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.  5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40  	Number lines, counting sticks and bar models should be used to show representation of counting in multiples.   	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25 , 30 $4 \times 3 = \square$

Y2 MULTIPLICATION

Objective & Strategy	Concrete	Pictorial	Abstract
Multiplication is commutative	Create arrays using counters and cubes and Numicon.    Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.  	Use representations of arrays to show different calculations and explore commutativity.  	12 = 3 × 4 12 = 4 × 3 Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15
Using the Inverse *This should be taught alongside division, so pupils learn how they work alongside each other.*			2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4 4 = 8÷ 2 Show all 8 related fact family sentences.

Y3

Pupils should be taught to:

- recall and use multiplication facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to written methods
- solve problems involving missing number problems involving multiplication including positive number scaling problems and correspondence problems where n objects are connected to m objects.

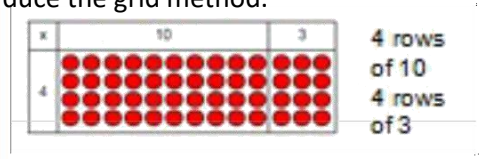
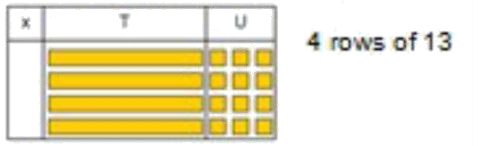
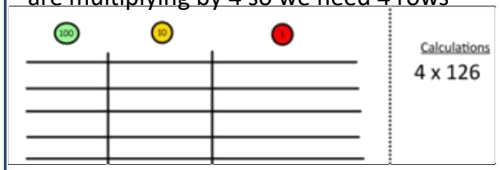
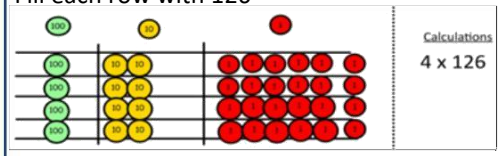
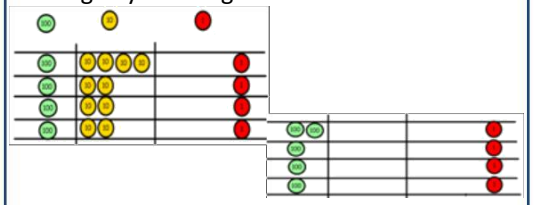
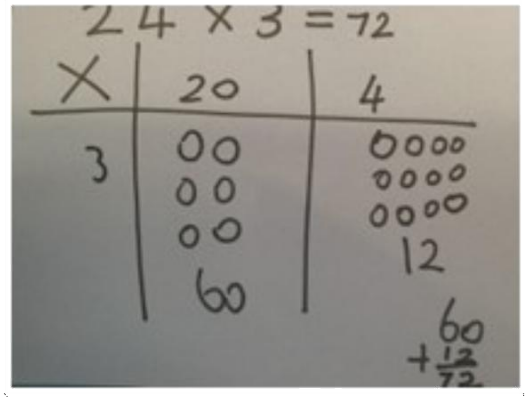
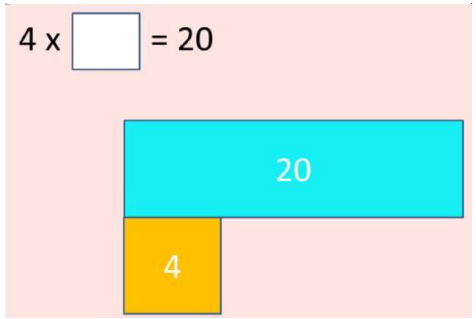
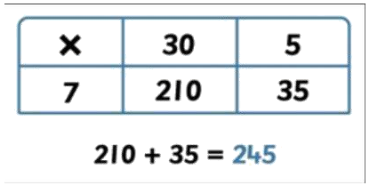
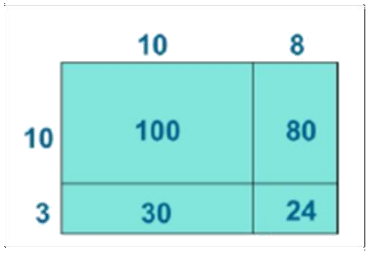
Pupils recall and use **2x 5x 10x 3x 4x 6x 8x and 9x**

Vocabulary

lots of, groups of, x, times, multiply, multiplication, product (new this year), multiplied by, multiple of, once, twice, three times, four times, five times... ten times... times as (big, long, wide and so on), repeated addition, array, row, column, double, halve, share, share equally

one each, two each, three each... group in pairs, threes... tens, equal groups of ÷, divide, divided by, divided into, left, left over

Y3 MULTIPLICATION

Objective & Strategy	Concrete	Pictorial	Abstract
Grid method	Show the links with arrays to first introduce the grid method.  Move onto base ten to move towards a more compact method.  Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows  Fill each row with 126  Add up each column, starting with the ones making any exchanges needed  Then you have your answer.	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.  Bar model are used to explore missing numbers 	Start with multiplying by one digit numbers and showing the clear addition alongside the grid.  Moving forward, multiply by a 2 digit number showing the different rows within the grid method.  Build on partitioning to develop grid multiplication e.g. *23 × 4*  = 92

Y4

Pupils should be taught to:

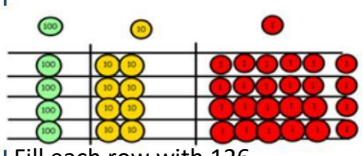
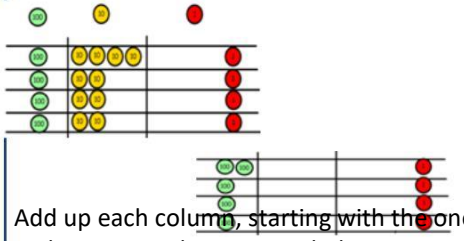
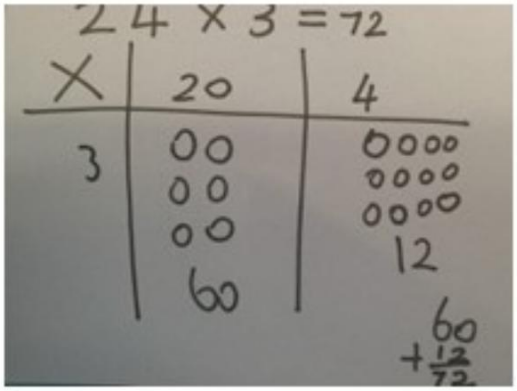
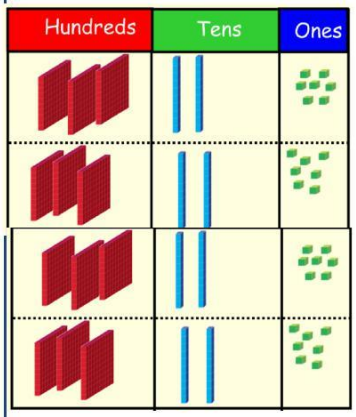
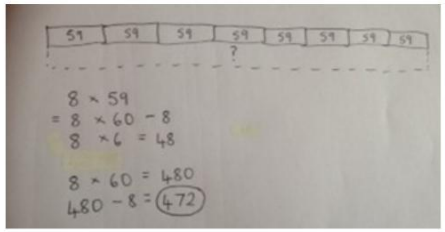
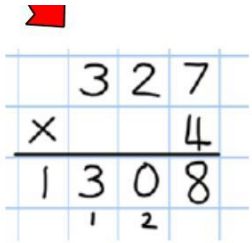
- recall and use multiplication facts for multiplication tables up to 12 x 12
- use place value, known and derived facts to multiply mentally, including: x0 x1 and multiplying together three numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying, including the distributive law to multiply two-digit numbers by one-digit including positive number scaling problems and correspondence problems where n objects are connected to m objects.

Pupils recall and use tables facts **up to 12 x 12**

Vocabulary

lots of, groups of, x, times, multiply, multiplication, product, multiplied by, multiple of, once, twice, three times, four times, five times... ten times... times as (big, long, wide and so on), repeated addition, array, row, column, double, halve, share, share equally, divisible by, factor, quotient, inverse

Y4 MULTIPLICATION

Objective & Strategy	Concrete	Pictorial	Abstract
Grid method recap from year 3 for 2 digits x 1 digit Move to multiplying 3 digit numbers by 1 digit. (year 4 expectation)	Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows  Calculations 4 x 126 Fill each row with 126  Add up each column, starting with the ones making any exchanges needed	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below. 	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. | × | 30 | 5 | | 7 | 210 | 35 | 210 + 35 = 245
Column multiplication	Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642  It is important at this stage that they always multiply the ones first. The corresponding long multiplication is modelled alongside	Use grid multiplication to multiply 3-digit numbers by 1-digit numbers e.g. 253 × 6 | × | 200 | 50 | 3 | | | 6 | 1200 | 300 | 18 | = 1518 The grid method may be used to show how this relates to a formal written method.  Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.	Use a vertical written algorithm (ladder) to multiply 3-digit numbers by 1-digit numbers e.g. 253 × 6 2 5 3 × 6 1 2 0 0 ← 6 × 200 3 0 0 ← 6 × 50 + 1 8 ← 6 × 3 1 5 1 8  This may lead to a compact method.

Y5

Pupils should be taught to:

- identify multiples and factors: all factor pairs of a number, common factors of two numbers, establish whether a number up to 100 is prime and recall prime numbers up to 19
- multiply numbers up to four digits by a one- or two-digit number using a formal written method
- multiply whole numbers and those involving decimals by 10, 100 and 1000.

Pupils recall and use tables facts **up to 12 x 12**

Vocabulary

lots of, groups of, x, times, multiply, multiplication, product, multiplied by, multiple of, once, twice, three times, four times, five times... ten times... times as (big, long, wide and so on), repeated addition, array, row, column, double, halve, share, share equally, divisible by, factor, quotient, inverse, long division/multiplication, short division/multiplication, divisor

Y6

Pupils should be taught to:

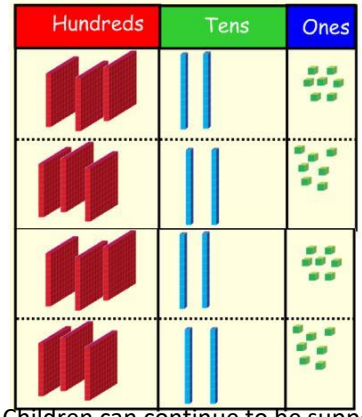
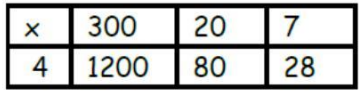
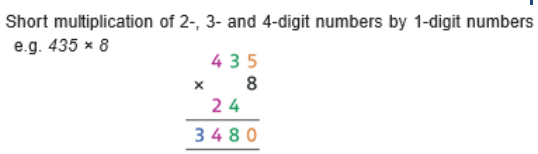
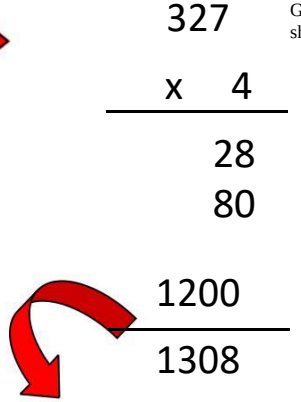
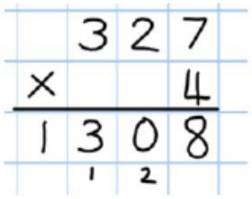
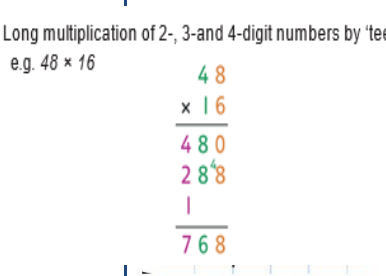
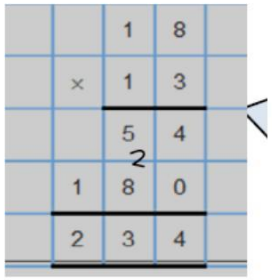
- identify multi-digit numbers up to 4 digits by a two-digit number using formal, long multiplication
- identify common factors, common multiples and common prime numbers
- use their knowledge of the order of operations to carry out calculations involving the four operations

Pupils recall and use tables facts **up to 12 x 12**

Vocabulary

lots of, groups of, x, times, multiply, multiplication, product, multiplied by, multiple of, once, twice, three times, four times, five times... ten times... times as (big, long, wide and so on), repeated addition, array, row, column, double, halve, share, share equally, divisible by, factor, quotient, inverse, long division/multiplication, short division/multiplication, divisor

Y5-6 MULTIPLICATION

Objective & Strategy	Concrete	Pictorial	Abstract
Column Multiplication for 3 and 4 digits x 1 digit.	 It is important at this stage that they always multiply the ones first. Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642	  	 Get through this to show compact.  This will lead to a compact method.
Column multiplication	Manipulatives may still be used with the corresponding long multiplication modelled alongside.	  Continue to use bar modelling to support problem solving	 18 x 3 on the first row (8 x 3 =24, carrying the 2 for 20, then 1 x 3) 18 x 10 on the 2nd row. Show multiplying by 10 by putting zero in units first

Y6 MULTIPLICATION

Objective & Strategy	Concrete	Pictorial	Abstract
Multiplying decimals up to 2 decimal places by a single digit.	Short multiplication of decimal numbers using × 100 and ÷ 100 e.g. *13·72 × 6 as (1372 × 6) ÷ 100 = 82·32* Short multiplication of money e.g. *£13·72 × 6* £13.72 × 6 241 £82.32 Grid multiplication of numbers with up to 2 decimal places by 1-digit numbers e.g. *6·76 × 4* × 6 0·7 0·06 4 24 2·8 0·24 = 27·04	Short multiplication of 2-, 3- and 4-digit numbers by 1-digit numbers e.g. *3743 × 6* 3743 × 6 421 22458 Long multiplication of 2-, 3- and 4-digit numbers by 2-digit numbers e.g. *456 × 38* 456 × 38 13680 3648 11 17328	Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer. 3·19 × 8 25·52 1 7

Division

YR

Pupils should be taught to:

Early Learning Goal Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing

Vocabulary

Share, half, double, equal, group, sort, set, match, halving is smaller, doubling is larger

	Teaching Points
Practically halving objects – both halves being exactly the same size - Start with play dough and things you can cut and then progress to practical objects. Eg: Pizza – cut in half to make two pieces – then add toppings , eg: 2 tomatoes - half of 2 is 1. Making number stories is a focus for understanding 	Counting and reading numbers to 20 Doubling using objects and numbers Halving using objects Sharing using objects Food as a focus for understanding

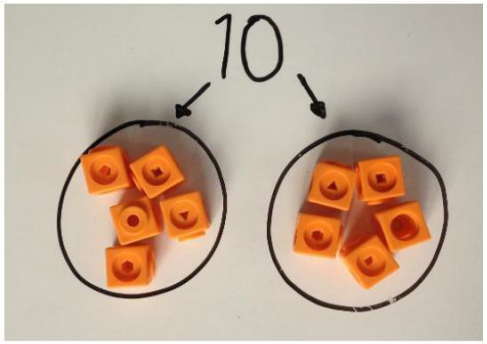
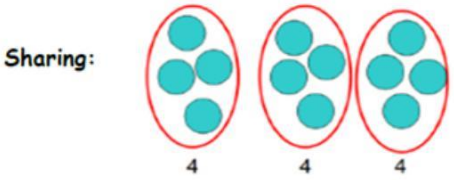
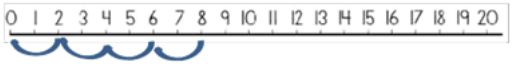
Y1

Pupils should be taught to:

- solve one-step problems involving division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Vocabulary

lots of, groups of x, times, multiply, multiplied by, once, twice, three times, four times, five times... ten times..., times as (big, long, wide and so on), repeated addition, array, row, column, double, halve, share, share equally, one each, two each, three each…

Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing	   I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  8 shared between 2 is 4  12 shared between 3 is 4 Sharing of 'chunks' begins to be modelled physically on a number line: 8 ÷ 2 = "How many 2s make 8?"  	12 shared between 3 is 4  Pupils begin to explore related division facts and linking these directly to inverse, commutative facts: 6 ÷ 2 = = 6 ÷ 2 6 ÷ = 3 3 = 6 ÷ ÷ 2 = 3 3 = ÷ 2 ÷ ∇ = 3 3 = ÷ ∇

Y1 DIVISION ÷

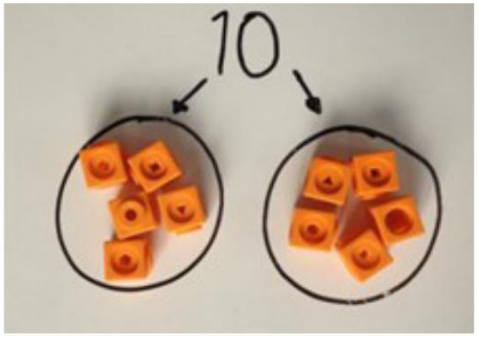
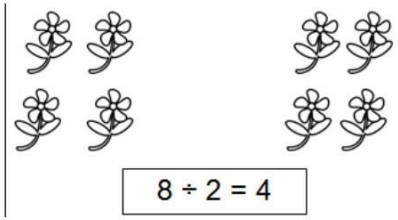
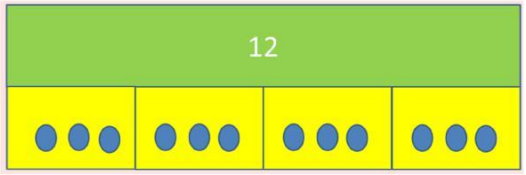
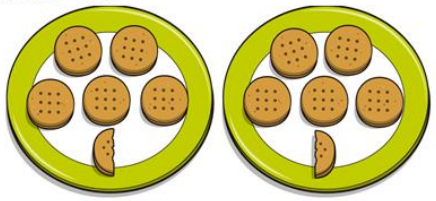
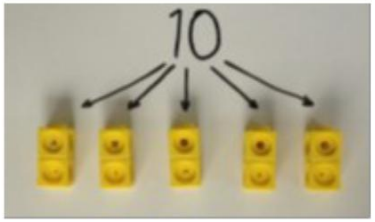
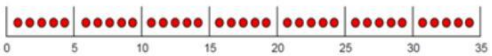
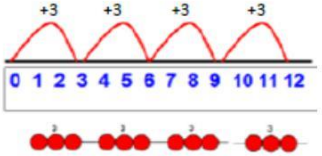
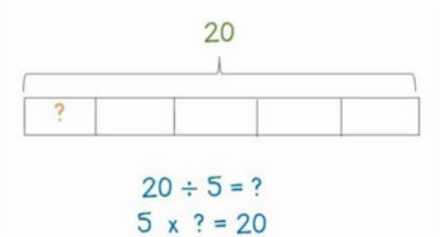
Y2

Pupils should be taught to:

- recall and use multiplication and division facts for the 2, 3, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for division within the multiplication tables and write them using the signs ÷ and =
- show that multiplication of two numbers is commutative but division is not
- solve problems involving division using materials, arrays, repeated addition, mental methods and division facts, including problems in contexts.

Vocabulary

Multiplication and division, lots of, groups of, x, times, multiply, multiplied by, multiple of, once, twice, three times, four times, five times... ten times... times as (big, long, wide and so on), repeated addition, array, row, column, double, halve, share, share equally one each, two each, three each... group in pairs, threes... tens, equal groups of, ÷, divide, divided by, divided, into, left, left over

Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing	 I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  Children use bar modelling to show and support understanding.  12 ÷ 4 = 3	12 ÷ 3 = 4 **Doubling and halving** Find half of numbers up to 40, including realising that half of an odd number gives a remainder of 1 or an answer containing a 1/2 e.g. *1/2 of 11 = 5 1/2* 
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  	Use number lines for grouping  12 ÷ 3 = 4 Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

Y2 DIVISION ÷

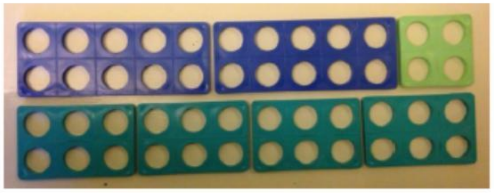
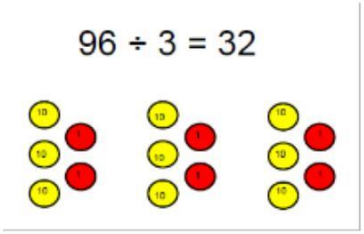
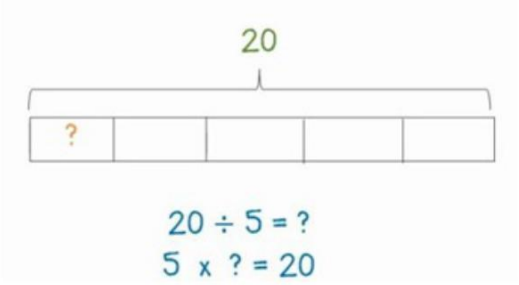
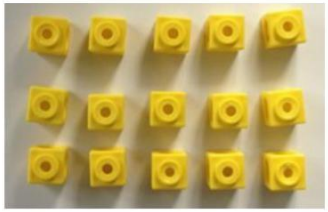
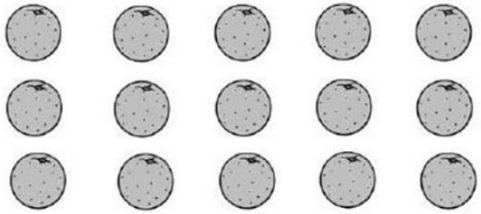
Y3

Pupils should be taught to:

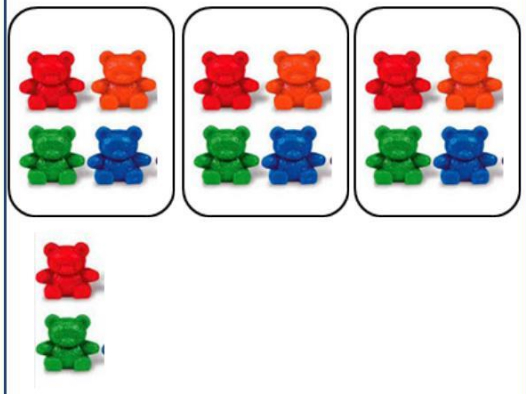
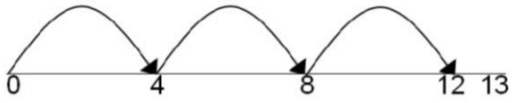
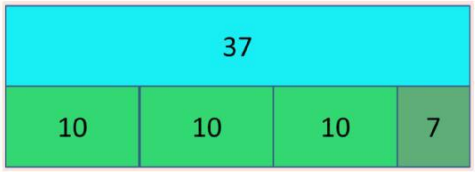
- recall and use multiplication and division facts for the 3, 4 and 8 x tables
- write and calculate mathematical statements for division using the multiplication tables they know, including 2-digit divided by 1-digit using mental and progressing to formal written methods
- solve problems, involving missing number problems, involving division, including positive number scaling problems and correspondence problems where n objects are connected to m objects.

Vocabulary

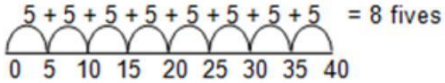
Multiplication and division, lots of, groups of, x, times, multiply, multiplied by, multiple of, once, twice, three times, four times, five times... ten times... times as (big, long, wide and so on), repeated addition, array, row, column, double, halve, share, share equally one each, two each, three each... group in pairs, threes... tens, equal groups of, ÷, divide, divided by, divided, into, left, left over, division, remainder

Objective & Strategy	Concrete	Pictorial	Abstract
Division as grouping	Use cubes, counters, objects or place value counters to aid understanding.  24 divided into groups of 6 = 4 	Continue to use bar modelling to aid solving division problems. 	How many groups of 6 in 24? 24 ÷ 6 = 4
Division with arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7 4 = 28 ÷ 7 7 = 28 ÷ 4

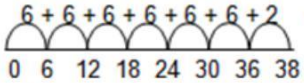
Y3 DIVISION ÷

Objective & Strategy	Concrete	Pictorial	Abstract
Division with remainders.	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  Use bar models to show division with remainders.  	Complete written divisions and show the remainder using r. 

Example without remainder:
40 ÷ 5
Ask "How many 5s in 40?"



Example with remainder:
38 ÷ 6



For larger numbers, when it becomes inefficient to count in single multiples, bigger jumps can be recorded using known facts.

Y4

Pupils should be taught to:

- recall multiplication and division facts up to 12 x 12
- use place value, known and derived facts to divide mentally, including dividing by 1
- solve problems involving dividing a three-digit number by one-digit and number using a formal layout

Vocabulary

multiplication and division, lots of, groups of, x, times, multiply, multiplied by, multiple of, product, once, twice, three times, four times, five times... ten times... times as (big, long, wide and so on), repeated addition, array, row, column, double, halve, share, share equally one each, two each, three each... group in pairs, threes... tens, equal groups of, ÷, divide, divided by, divided into, divisible by, left, left over, division, remainder, factor, quotient, inverse

Y5

Pupils should be taught to:

- identify multiples and factors, including finding all factor pairs of a number, common factors of two numbers, know and use the vocabulary of prime numbers and establish whether a number up to 100 is prime
- multiply and divide numbers mentally drawing on known facts
- divide numbers up to 4 digits by a one-digit number using a written method and interpret remainders appropriately for the context
- divide whole numbers and those involving decimals by 10, 100 and 1000.

Vocabulary

multiplication and division, lots of, groups of, x, times, multiply, multiplied by, multiple of, product, once, twice, three times, four times, five times... ten times... times as (big, long, wide and so on), repeated addition, array, row, column, double, halve, share, share equally one each, two each, three each... group in pairs, threes... tens, equal groups of, ÷, divide, divided by, divided into, divisible by, left, left over, division, remainder, factor, quotient, inverse, long and short division

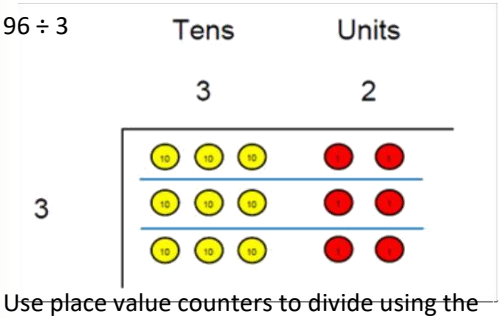
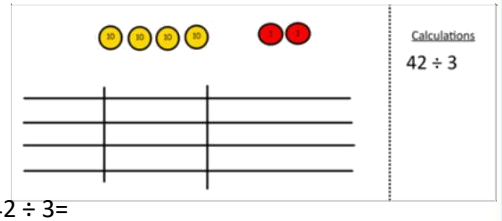
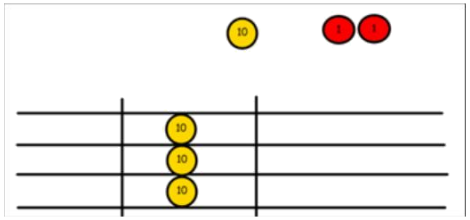
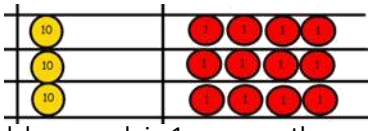
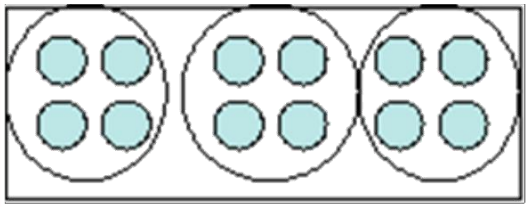
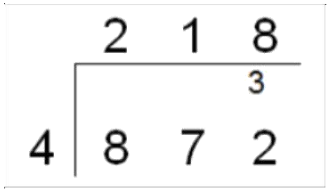
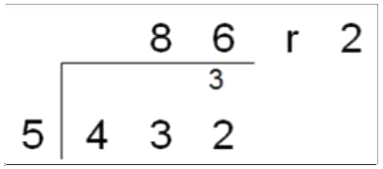
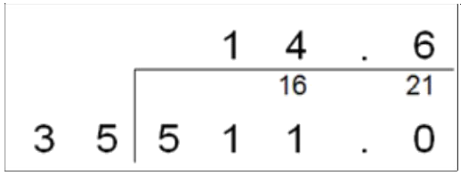
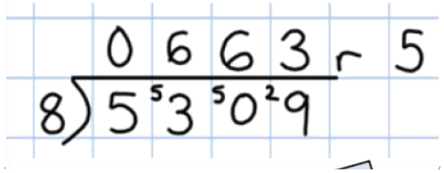
Y6

Pupils should be taught to:

- divide numbers up to 4 digits by a two-digit number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding as appropriate for the context.
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division as appropriate.

Vocabulary

multiplication and division, lots of, groups of, x, times, multiply, multiplied by, multiple of, product, once, twice, three times, four times, five times... ten times... times as (big, long, wide and so on), repeated addition, array, row, column, double, halve, share, share equally one each, two each, three each... group in pairs, threes... tens, equal groups of, ÷, divide, divided by, divided into, divisible by, left, left over, division, remainder, factor, quotient, inverse, long and short division , divisor

Objective & Strategy	Concrete	Pictorial	Abstract
Divide at least 3 digit numbers by 1 digit. Short Division	96 ÷ 3  Use place value counters to divide using the bus stop method alongside  42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder.  Move onto divisions with a remainder.  Finally move into decimal places to divide the total accurately.  

Long Division

Y6

DIVISION ÷

Step 1—a remainder in the ones

```
   h t o
   0 4 1 R1
  ---------
4 ) 1 6 5
```

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

```
     th h t o
     0 4 0 0 R7
    -----------
8 ) 3 2 0 7
```

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times (3,200 ÷ 8 = 400)
8 goes into 0 zero times (tens).
8 goes into 7 zero times, and leaves a remainder of 7.

Long Division

Y6

DIVISION

Step 1 continued...

```
    h t o
    0 6 1
  ┌──────
4 ) 2 4 7
    - 4
    ---
      3
```

When dividing the ones, 4 goes into 7 one time. Multiply 1 × 4 = 4, write that four under the 7, and subract. This finds us the remainder of 3.

Check: 4 × 61 + 3 = 247

```
     th h t o
     0 4 0 2
  ┌──────────
4 ) 1 6 0 9
    - 8
    -----
      1
```

When dividing the ones, 4 goes into 9 two times. Multiply 2 × 4 = 8, write that eight under the 9, and subract. This finds us the remainder of 1.

Check: 4 × 402 + 1 = 1,609

Long Division

Y6 DIVISION

Step 2—a remainder in the tens

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 2)5 8	t o 2 2)5 8 -4 1	t o 2 9 2)5 8 -4 ↓ 1 8
Two goes into 5 two times, or 5 tens ÷ 2 = 2 whole tens -- but there is a remainder!	To find it, multiply 2 × 2 = 4, write that 4 under the five, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 9 2)5 8 -4 1 8	t o 2 9 2)5 8 -4 1 8 -1 8 0	t o 2 9 2)5 8 -4 1 8 -1 8 0
Divide 2 into 18. Place 9 into the quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract.	The division is over since there are no more digits in the dividend. The quotient is 29.

Long Division

Step 2—a remainder in any of the place values

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
h t o 1 2)2 7 8 Two goes into 2 one time, or 2 hundreds ÷ 2 = 1 hundred.	h t o 1 2)2 7 8 -2 0 Multiply 1 × 2 = 2, write that 2 under the two, and subtract to find the remainder of zero.	h t o 1 8 2)2 7 8 -2 ↓ 0 7 Next, drop down the 7 of the tens next to the zero.
Divide.	**Multiply & subtract.**	**Drop down the next digit.**
h t o 1 3 2)2 7 8 -2 0 7 Divide 2 into 7. Place 3 into the quotient.	h t o 1 3 2)2 7 8 -2 0 7 - 6 1 Multiply 3 × 2 = 6, write that 6 under the 7, and subtract to find the remainder of 1 ten.	h t o 1 3 2)2 7 8 -2 0 7 - 6 1 8 Next, drop down the 8 of the ones next to the 1 leftover ten.
1. Divide.	**2. Multiply & subtract.**	**3. Drop down the next digit.**
h t o 1 3 9 2)2 7 8 -2 0 7 - 6 1 8 Divide 2 into 18. Place 9 into the quotient.	h t o 1 3 9 2)2 7 8 -2 0 7 - 6 1 8 -1 8 0 Multiply 9 × 2 = 18, write that 18 under the 18, and subtract to find the remainder of zero.	h t o 1 3 9 2)2 7 8 -2 0 7 - 6 1 8 -1 8 0 There are no more digits to drop down. The quotient is 139.

Y6

DIVISION ÷